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A HIGHLY EFFICIENT M/G/∞ GENERATOR OF SELF-SIMILAR TRACES

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ABSTRACT

Several traffic measurements reports have convincingly shown the presence of self-similarity in current networks, inducing a revolution in the stochastic modeling of traffic. The essence of this behavior can be captured by several classes of self-similar processes. But the use of these processes in performance analysis has opened new problems and research issues in simulation studies, where the efficient generation of synthetic sample paths with self-similar properties is one of the main topics. In this paper, we present an M/G/∞ generator of self-similar traces, based on a highly efficient simulation model using the decomposition property of Poisson processes and the memoryless property of geometric random variables.

1 INTRODUCTION

Traffic modeling has traditionally been based almost exclusively on the assumption of independence between the random variables that describe arrivals to a network. The fundamental reason has been the analytical tractability. However, several traffic measurements studies (Leland et al. 1994, Beran et al. 1995, Paxson and Floyd 1995) have convincingly shown the presence of self-similarity in modern networks, involving long-range correlations over arbitrarily large time scales, a phenomenon usually referred to as Long-Range Dependence (LRD). All these findings have contributed to a very important revolution in the stochastic modeling of traffic, since the impact of the correlation on the performance metrics may be drastic (Norros 1994, Erramilli et al. 1996), and the validity of traditional processes, like Markovian or autoregressive, is in doubt because modeling LRD through these processes requires many parameters, whose interpretation becomes difficult. Because of this, the use of self-similar processes for network traffic modeling purposes is essential, due to their capability to exhibit LRD over all time scales by making use of few parameters (parsimonious modeling).

The application of self-similar processes in network simulation studies has opened a wide range of research topics dealing with new problems. One of the most important issues is the synthetic generation of sample paths of LRD processes, since real traces collected by measurements are of limited length and lack the diversity required to make flexible enough simulation studies.

An interesting self-similar process is the occupancy process of an M/G/∞ queueing model, referred to as the M/G/∞ process. Besides its use in analytical studies (Duffield 1987, Tsoukatos and Makowski 1997), it has been used in simulation studies (Krunz and Makowski 1998, Poon and Lo 2001), where it has several important advantages, such as the possibility of on-line generation. Furthermore, there exists a trivial method of producing exact sample paths of the process with complexity $\mathcal{O}(n)$: to simulate the M/G/∞ queue, sampling the occupancy of the system at integer instants. Varying the service time distribution many forms of time dependence can be obtained, which makes this process a good candidate for modeling many types of correlated traffic. In (Súarez et al. 2002) the authors present a discrete random variable whose distribution (S distribution) is heavy-tailed with two parameters, a feature that enables the modeling of both the short-term and the long-term correlation behavior of the resulting M/S/∞ process.

The tail of the marginal distribution plays an important role in performance evaluation (Grossglauser and Bolot 1996). To model the empirical marginal distribution of some real sequences we need to transform the Poisson marginal distribution of the M/G/∞ process into a more appropriate heay-tailed form, and small values of the arrival rate λ of the Poisson input process are inappropriate for the transformation process (Poon and Lo 2001). On the other hand, the complexity of the generator is an increasing function of λ, that is directly related to the mean of the M/G/∞ process. In order to get a highly efficient simulation model, able to deal with a wide range of input parameters, in this paper we propose the use of the decomposition property of Poisson processes, and the memoryless property of geometric random variables.

The remainder of the paper is organized as follows. We begin reviewing the main concepts related to LRD and statistical self-similarity in Section 2. In Section 3 we present the M/G/∞ process and the S distribution that we use to model the service time. In Section 4 we describe the method that we propose in order to improve the efficiency of the generator of samples of the M/G/∞ process, and evaluate the improved simulation model applied to the sample generation of the M/S/∞ process. Finally, Section 5 summarizes the conclusions.

2 LRD AND SELF-SIMILARITY

It is said that a process exhibits LRD when its autocorrelation function is not summable, i.e., $\sum_{k=1}^{\infty} \mathrm{r}_k = \infty$, like in those processes whose autocorrelation function decays hyperbolically:

$$\exists \beta \in (0,1) \left| \lim_{k\to\infty} \frac{\mathrm{r}_k}{k^{-\beta}} = \mathsf{c}_\mathrm{r} \in (0,\infty) \right. . \qquad (1)$$

Let $X = \{X_k; k = 1,2,\dots\}$ be a stationary stochastic process with finite variance and let $X^{(m)}$ be the corresponding aggregated process (with aggregation level m), obtained by averaging the original sequence X over non-overlapping blocks of size m, $X^{(m)} = \{\overline{X_i}[m]; i = 1,2,\dots\}$, where $\overline{X_i}[m] = \frac{1}{m} \cdot \sum_{j=(i-1)\cdot m+1}^{i\cdot m} X_j$.

The process X is called exactly second-order self-similar, with self-similarity parameter H (Hurst 1951), if the aggregated process $X^{(m)}$ scaled by $m^{1-\mathsf{H}}$ has the same variance and autocorrelation as X for all m, that is, if the aggregated processes possess the same nondegenerate correlation structure as the original process.

The autocorrelation function of X and $X^{(m)}$ is:

$$\mathrm{r}_k = \frac{1}{2} \cdot \left[(k+1)^{2\mathsf{H}} - 2k^{2\mathsf{H}} + (k-1)^{2\mathsf{H}} \right] \quad \forall k \geq 1, \qquad (2)$$

where $\lim_{k\to\infty} \frac{\mathrm{g}_k^{\mathsf{H}}}{k^{2\mathsf{H}-2}} = \mathsf{H} \cdot (2\mathsf{H}-1)$, that is, it decays hyperbolically as in (1), and so the process exhibits LRD (Cox 1984).

If (2) is satisfied asymptotically by the autocorrelation function of the aggregated process, $\mathrm{r}_k^{(m)}$, then the process is called asymptotically second-order self-similar: $\lim_{m\to\infty} \mathrm{r}_k^{(m)} = \mathrm{g}_k^{\mathsf{H}} \quad \forall k \geq 1$.

It has been shown that a covariance stationary process whose autocorrelation function decays hyperbolically is asymptotically second-order self-similar (Tsybakov and Georganas 1997). For this reason, although LRD and self-similarity are not equivalent concepts, they are often utilized without distinction.

3 M/G/∞ PROCESS

An interesting self-similar process is the occupancy process of an M/G/∞ queueing model. In such model, customers arrive according to a Poisson process with rate λ to a system of infinitely many servers, and their service times constitute a sequence of continuous independent and identically distributed random variables distributed as the general random variable S with finite mean value $\mathrm{E}[S]$. The number of customers, or busy servers, in the system at any instant t, $\{X(t); t \in \Re^+\}$, has a Poisson marginal distribution with mean value $\lambda \, \mathrm{E}[S]$. We are interested on the discrete-time version of $\{X(t); t \in \Re^+\}$, that is: $X \triangleq \{X_i \triangleq X(i); i = 1,2,\dots\}$, a stochastic process referred to as the M/G/∞ process. The most natural approach to generate an M/G/∞ process is to simulate the queue in discrete-time, since its simulation will be more efficient (Suárez et al. 2002).

3.1 Discrete-Time Model

Let $A = \{A_n; n = 1,2,\dots\}$ be a renewal stochastic process, where A_n is a Poisson random variable with mean value λ and represents the number of arrivals at instant n; let $\{\{S_{n,i}; i = 1,\dots,A_n\}; n = 1,2,\dots\}$ be a renewal stochastic process where $S_{n,i}$ is distributed as a positive-valued discrete random variable S with finite mean value $\mathrm{E}[S]$, and corresponds to the service time of the i-th arrival at instant n.

If the initial number of users X_0 is a Poisson random variable of mean value $\lambda \cdot \mathrm{E}[S]$ and their service times $\{\widehat{S}_j; j = 1,\dots,X_0\}$ are mutually independent and have the same distribution as the residual life of S, then the stochastic process $X = \{X_n; n = 1,2,\dots\}$ is strict-sense stationary and ergodic, and enjoys equivalent properties to those of the original continuous-time M/G/∞ process:

1. the process X has a Poisson marginal distribution and mean value:

$$\mu \triangleq \mathrm{E}[X] = \lambda \cdot \mathrm{E}[S], \qquad (3)$$

2. its autocorrelation function is given by:

$$\mathrm{r}_k = 1 - \frac{\sum_{i=0}^{k-1} \Pr[S > i]}{\mathrm{E}[S]} \quad \forall k = 1,2,\dots \qquad (4)$$

3. it exhibits LRD $\Longleftrightarrow \mathrm{Var}[S] = \infty$, as it may happen in heavy-tailed service distributions.

3.2 S Distribution

In Suárez et al. (2002) the authors propose to use the S random variable, having its mean value and the cdf of its residual life the explicit expressions given in Appendix B. Its main characteristic is that of being a heavy-tailed distribution with two parameters, a feature that enables the modeling of both short-term and long-term correlation behavior of the occupany process.

The autocorrelation function of the resulting M/S/∞ process is:

$$\mathrm{r}_k = \begin{cases} 1 - \frac{\alpha-1}{\mathsf{m}\alpha} \cdot k & \forall k \in (0, \mathsf{m}] \\ \frac{1}{\alpha} \cdot \left(\frac{\mathsf{m}}{k}\right)^{\alpha-1} & \forall k \geq \mathsf{m}. \end{cases}$$

Given the three desired parameters of the process X (mean value μ, Hurst parameter H and one-lag autocorrelation coefficient r_1) the parameters of the M/S/∞ model can be computed as follows:

$$\alpha = 3 - 2\mathsf{H} \tag{5}$$

$$\mathsf{m} = \begin{cases} (\alpha \mathrm{r}_1)^{\frac{1}{\alpha-1}} & \forall \mathrm{r}_1 \in \left(0, \frac{1}{\alpha}\right] \\ \frac{\frac{\alpha-1}{\alpha}}{1-\mathrm{r}_1} & \forall \mathrm{r}_1 \in \left[\frac{1}{\alpha}, 1\right) \end{cases}$$

$$\lambda = \begin{cases} \mu \cdot \frac{\alpha \mathsf{m} - \mathsf{m}^\alpha}{\mathsf{m}\alpha} & \forall \mathsf{m} \in (0, 1] \\ \mu \cdot \frac{\alpha-1}{\mathsf{m}\alpha} & \forall \mathsf{m} \geq 1. \end{cases}$$

4 A HIGHLY EFFICIENT M/G/∞ GENERATOR

The M/G/∞ process has Poisson marginal distribution, whose tail drops faster than that of the empirical marginal distribution of some real sequences. The tail of the marginal distribution plays an important role in performance evaluation (Grossglauser and Bolot 1996). Therefore, we need to transform the Poisson marginal distribution of the M/G/∞ process into a more appropriate one, but this introduces an efficiency problem. On the one hand, small values of the arrival rate λ of the Poisson input process are inappropriate for the transformation process but, on the other hand, the complexity of the generator is a linear increasing function of λ.

In order to improve the efficiency for large mean values of the M/G/∞ process, that given equation (3) implies also large values of λ, we propose to use the decomposition property of Poisson processes and the memoryless property of geometric random variables.

4.1 Description of the Proposed Method

When we use a discrete-time simulation model of the M/G/∞ system, every sample value X_n requires the generation of one sample of the Poisson random variable A_n, with mean value λ, and A_n samples of the random variable S. We denote by N the mean number of random values that have to be generated for each sample value of the occupancy process. In this case $\mathsf{N} = \lambda + 1$. For large values of λ, the computational time can be very high. In Table 1 we can see the mean number of random values for different values of the parameters of the M/S/∞ process. We denote this method Direct Method (DM).

Table 1: DM. $\mathsf{N} = \lambda + 1$

M/S/∞ parameters		$\mathsf{H} = 0.6$	$\mathsf{H} = 0.9$
$\mathrm{r}_1 = 0.1$	$\mu = 64$	58.6	58.6
	$\mu = 1024$	922.6	922.6
	$\mu = 16384$	14746.6	14746.6
$\mathrm{r}_1 = 0.9$	$\mu = 64$	7.4	7.4
	$\mu = 1024$	103.4	103.4
	$\mu = 16384$	1639.4	1639.4

In order to reduce N, in Sousa et al. (2002) we have proposed the use of the decomposition property of Poisson process, that motivates the Simple Composition Method (SCM). In this work we propose an improvement of this method using the memoryless property of geometric random variables. We denote the improved method Mixed Composition Method (MCM). This method has to guarantee that the number of arrivals at each instant n demanding k units of service time is a Poisson random variable with mean value $\lambda \cdot \Pr[S = k]$.

Before explaining and evaluating the MCM, in the following section we show a very efficient method to deal with the departure times in an M/G/∞ system with geometric distribution for the service process.

4.1.1 Service Time with Geometric Distribution

Considering an M/G/∞ queueing system, whose arrival process is Poissonian with rate λ, and with geometric distribution for the service process G, with parameter p, in a discrete-time simulation every sample of the occupancy process, X_n, is obtained after adding the instantaneous arrivals, A_n, and subtracting the departures happening at instant n, L_n.

Using the memoryless property of geometric random variables we divide the users in the system at each instant n, X_n, in two groups: those which are going to leave at instant $n+1$, that we can generate as a sample of a binomial random variable, B, with parameters p and X_n, L_{n+1}, and the rest, $X_n - L_{n+1}$.

If the service time follows a shifted geometric random variable, $G^{'} = \mathsf{K} + G$, an array of size K is needed in order to delay the departures K units of time and, at each instant n, L_{n+1} can be generated as a sample of a binomial random variable B, with parameters p and $X_n - D_n$, where D_n is the number of users in the array of delays.

Figures 1 and 2 illustrate these situations. In any case, we reduce the mean number of random values per sample of X_n from $1+\lambda$ to 2 (A_n and L_{n+1}).

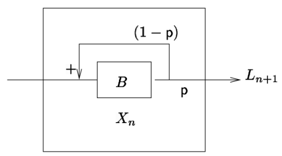

Figure 1: Geometric Service Time

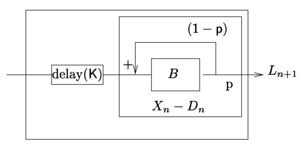

Figure 2: Shifted Geometric Service Time

4.1.2 Mixed Composition Method

In this method, we divide the arrivals at each instant n into $\mathsf{K}+\mathsf{L}+1$ groups, according to the random variable from which their service times are generated.

For the K first groups, the mean number of arrivals at each group is $\lambda \cdot \mathsf{d}_i; i = 1,2,\ldots,\mathsf{K}$, being $\mathsf{d}_i = \Pr[S=i]; i = 1,2,\ldots,\mathsf{K}$, and the service times are deterministic, with values $i = 1,2,\ldots,\mathsf{K}$, as proposed (Sousa et al. 2002).

For the L following groups, we fit the distribution of the S random variable with the composition of the distributions of L geometric random variables, with parameters $\mathsf{p}_i; i = 1,2,\ldots,\mathsf{L}$ and shifted to $k = \mathsf{K}+1$, $G_i^{'} = \mathsf{K} + G_i; i = 1,2,\ldots,\mathsf{L}$. We denote by $\mathsf{g}_i^{'}; i = 1,2,\ldots,\mathsf{L}$ the composition factors. The adjustment has to be such that: $\Pr[S=k] \geq \sum_{i=1}^{\mathsf{L}} \mathsf{g}_i^{'} \cdot \Pr\left[G_i^{'} = k\right] \quad \forall k > \mathsf{K}$. Since each shifted geometric random variable has two free parameters, $\mathsf{p_i}$ and the composition factor $\mathsf{g}_i^{'}$, we choose them looking for the equality in $2\mathsf{L}$ points. The mean number of arrivals at each of these L groups is $\lambda \cdot \mathsf{g}_i^{'}; i = 1,2,\ldots,\mathsf{L}$. We are interested in maximizing $\sum_{i=1}^{\mathsf{L}} \mathsf{g}_i^{'}$, because obtaining their departure times from binomial random variables, $B_i; i = 1,2,\ldots,\mathsf{L}$, is very efficient, as explained in Section 4.1.1.

Finally, we denote by R the random variable obtained from the difference between $\Pr[S=k]$ and $\sum_{i=1}^{\mathsf{L}} \mathsf{g}_i^{'} \cdot \Pr[G_i^{'} = k] \quad \forall k > \mathsf{K}$, where r is the probability that a service time is generated from this random variable. For the last group, the mean number of arrivals is $\lambda \cdot \mathsf{r}$.

As we can see in Figure 3, using the decomposition property of Poisson processes we divide the original input process into $\mathsf{K}+\mathsf{L}+1$ Poisson processes.

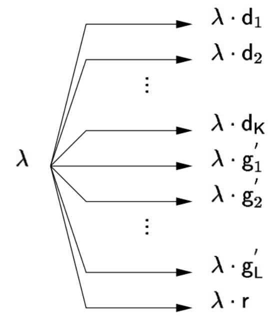

Figure 3: MCM: The Arrivals Input Process

With this method, each sample value X_n requires the generation of:

- one sample for each Poisson random variable $A_{n,i}; i = 1,2,\ldots,\mathsf{K}$, with mean value $\lambda \cdot \mathsf{d}_i$ for $i = 1,2,\ldots,\mathsf{K}$,
- one sample for each Poisson random variable $A_{n,i}; i = \mathsf{K}+1, \mathsf{K}+2, \ldots, \mathsf{K}+\mathsf{L}$, with mean value $\lambda \cdot \mathsf{g}_i^{'}$ for $i = 1,2,\ldots,\mathsf{L}$,
- one sample for each binomial random variable B_i for $i = 1,2,\ldots,\mathsf{L}$,
- one sample of the Poisson random variable $A_{n,\mathsf{K}+\mathsf{L}+1}$, with mean value $\lambda \cdot \mathsf{r}$,
- $A_{n,\mathsf{K}+\mathsf{L}+1}$ samples of the random variable R.

The mean number of random values that have to be generated for each sample value of the occupancy process is $\mathsf{N} = \mathsf{K} + 2\mathsf{L} + 1 + \lambda \cdot \mathsf{r}$. Since our aim is to minimize this quantity, and now the best value of L is determined by the best value of K, we begin with K as in the SCM and we decrease it up to obtain the minimum value of N.

Memory Requirements: If we denote by $X_{n,G_i^{\prime}}; i = 1,2,\ldots,\mathsf{L}$ the number of users in the system that come from the second group of arrivals, at each instant n we need to store, for each shifted geometric random variable, the number of users that are going to remain in the system at instant $n+1$, $X_{n,G_i^{\prime}} - L_{n+1,G_i^{\prime}}$, where $L_{n+1,G_i^{\prime}}$ can be generated as a sample of the binomial random variable B_i, with parameters $\mathsf{p_i}$ and $X_{n,G_i^{\prime}}$.

Moreover, since the range of the shifted geometric random variables begins at $k = \mathsf{K}+1$, we need for each one an array of size $\mathsf{K}+1$, in order to delay the departures $\mathsf{K}+1$ units of time and, at each instant n, the number of users that are going to leave at instant $n+1$ are generated from $X_{n,G_i'}$ - $D_{n,G_i'}$, being $D_{n,G_i'}$ the number of users in the array of delays of the i^{th} shifted geometric random variable.

4.2 Evaluation on the M/S/∞ Process

We evaluate this improved simulation model with the M/S/∞ process. In the remaining of this section we check the effect of the input parameters (μ, H and r_1) on the mean number of random values per sample value X_n, N. We vary the mean value of the M/S/∞ process in powers of two, $\mu = 2^m$, and fix one of the two remaining input parameters to a moderate value ($r_1 = 0.5$, $\mathsf{H} = 0.7$), while using a set of values for the other one ($\mathsf{H} \in \{0.6, 0.7, 0.8, 0.9\}$, $r_1 \in \{0.1, 0.5, 0.9\}$): the higher (lower) values for r_1 and H are meant to be representative of strong (weak) correlation and of strong (weak) LRD behavior.

First, we show the effect of the mean value of the M/S/∞ process and the Hurst parameter on N in Figure 4 (left). With respect to μ, we observe that N is several orders of magnitude lower than μ in the studied interval, so the method is very efficient even for large values of μ. On the other hand, we can see that asymptotically N seems to be an increasing function of H for every μ. It is the result of the higher dispersion of the service time distribution for lower α (higher H) values when $\mathrm{E}[\mathrm{S}]$ is constant (same r_1 implies same $\mathrm{E}[\mathrm{S}]$, from (4)).

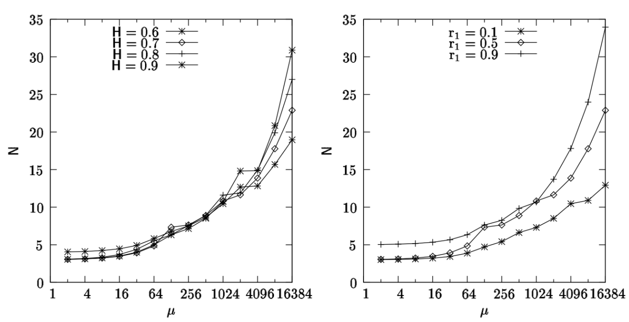

Figure 4: MCM: N for $r_1 = 0.5$ (Left) and $\mathsf{H} = 0.7$ (Right)

Figure 4 (right) shows the same behavior of N with respect to μ. Nevertheless, the effect of the parameter r_1 on N is clearly stronger than that of H, being an increasing function of r_1 for large values of μ. This behavior is consequence of a constant dispersion of S (same H implies same α, from (5)) and $\mathrm{E}[\mathrm{S}]$ increases as r_1 does, from (4) and λ decreases as $\mathrm{E}[\mathrm{S}]$ increases, from (3).

The number of geometric random variables is an increasing function of μ, but it is important to stand out that its highest value is $\mathsf{L} = 3$, for all the combinations of H and r_1 in the studied range of μ.

In Figure 5 we show the number of deterministic and geometric random variables, K and L, obtained with the MCM, when $\mathsf{H} = 0.7$ and $r_1 = 0.5$, as well as the mean number of random variables, N.

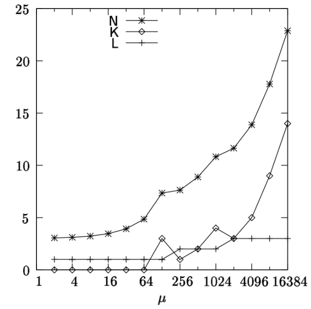

Figure 5: $\mathsf{N}, \mathsf{K}, \mathsf{L}$ for $\mathsf{H} = 0.7$ and $r_1 = 0.5$

As we can see, when $\mu = 1024$, $\mathsf{H} = 0.7$ and $r_1 = 0.5$, the number of deterministic random variables is 4, the number of geometric random variables is 2 and $\mathsf{N} = 10.82$. For each geometric random variable the distribution of the S random variable is fitted in two points. In this case, 5, 6, 15 and 16. In Figure 6 we observe the distribution of the S random variable, $p[k] = p_{\mathrm{S}}[k] = \Pr[\mathrm{S} = k]$, and the distribution of the G_1' and G_2' random variables scaled by the composition factors g_1' and g_2', $p_1[k] = \mathrm{g}_1' \cdot p_{\mathrm{G}_1'}[k]$, $p_2[k] = \mathrm{g}_2' \cdot p_{\mathrm{G}_2'}[k]$ and $p_{1\_2}[k] = p_1[k] + p_2[k]$. And in Table 2 we show the decomposition of the arrivals input process for this example.

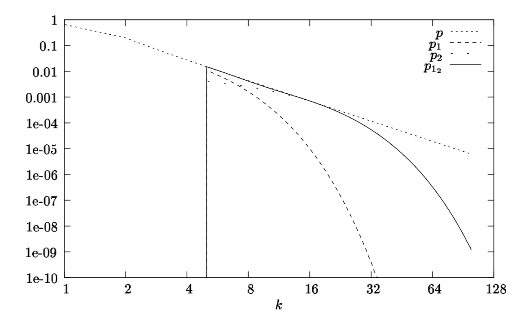

Figure 6: Distributions for $\mathsf{H} = 0.7$ and $r_1 = 0.5$

Table 2: The Arrivals Input Process for $\mu = 1024$, $\mathsf{H} = 0.7$ and $r_1 = 0.5$

$\lambda = 512$						
$\lambda \cdot \mathsf{d}_1$	$\lambda \cdot \mathsf{d}_2$	$\lambda \cdot \mathsf{d}_3$	$\lambda \cdot \mathsf{d}_4$	$\lambda \cdot \mathsf{g}_1'$	$\lambda \cdot \mathsf{g}_2'$	$\lambda \cdot \mathsf{r}$
337.79	101.26	30.96	14.06	11.99	14.11	1.82

Finally, in Table 3 we compare the mean number of random values, N, obtained with each method for different values of the input parameters, H and r_1, and mean value $\mu = 16384$ of the M/S/∞ process. If the reduction with the SCM was already substantial, with the MCM it is even greater.

Table 3: N for $\mu = 16384$

Method	$\mathsf{H} = 0.6$, $\mathrm{r}_1 = 0.1$	$\mathsf{H} = 0.9$, $\mathrm{r}_1 = 0.1$	$\mathsf{H} = 0.6$, $\mathrm{r}_1 = 0.9$	$\mathsf{H} = 0.9$, $\mathrm{r}_1 = 0.9$
DM	14746.6	14746.6	1639.4	1639.4
SCM	25.4	28.2	67.9	76.6
MCM	12.5	13.8	27.9	45.9

Table 4: MCM: CPU Time Improvement

M/S/∞ parameters		$\mathsf{H} = 0.6$	$\mathsf{H} = 0.9$
$\mathrm{r}_1 = 0.1$	$\mu = 4096$	32.2	35.6
	$\mu = 8192$	53	57.6
	$\mu = 16384$	97.6	97.2
$\mathrm{r}_1 = 0.9$	$\mu = 4096$	3.3	3.3
	$\mu = 8192$	6.2	4.6
	$\mu = 16384$	9.7	6.1

4.3 Time Measurements

In this section, we measure the real performance of our implementation of the proposed simulation model (briefly commented in Appendix A). We measured the running times in seconds (using the Unix command `time`) for the generation of sample paths of $n = 2 \cdot 10^6$ values of the M/S/∞ process in a 4-CPU Xeon 2 machine.

First, in Figure 7, we show the improvement regarding to the DM (Suárez et al. 2002):

$$\mathsf{R} = \frac{\text{running time of DM}}{\text{running time of MCM}}.$$

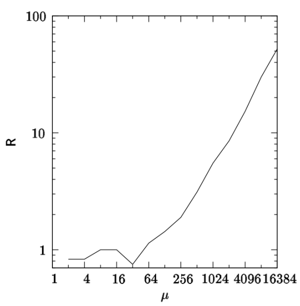

Figure 7: MCM: CPU Time Improvement for $\mathsf{H} = 0.7$ and $\mathrm{r}_1 = 0.5$

We observe that the efficiency of the generator based on the MCM is practically equal to that of the original method for small values of μ, but significantly better as μ increases. This is mainly due to the reduction in the number of samples that we have to generate for each sample value of the occupancy process: $\mathsf{N} = \lambda + 1$ with the M/S/∞ process and $\mathsf{N} = \mathsf{K} + 2\mathsf{L} + 1 + \lambda \cdot \mathsf{r}$ with the compound method.

In Table 4 we can see the improvement for different values of the input parameters of the M/S/∞ process.

Finally, in Figure 8 we observe the effect of μ and H (left) or r_1 (right) on the running time of the simulation model improved with the MCM.

Compared to Figure 4, we can see how the running time behaves as a function of the mean number of random values per sample value of the M/S/∞ process, N, as expected, although it also depends of the different types (and complexity) and mean values of the random variables

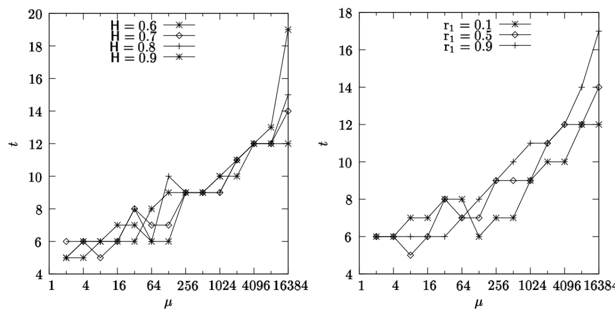

Figure 8: MCM: CPU Time for $\mathrm{r}_1 = 0.5$ (Left) and $\mathsf{H} = 0.7$ (Right)

(binomials, Poissonians and the *R* random variable) from which the random values are generated. We can see that the time is an increasing function of μ (and so of λ) but the complexity is sub $\mathcal{O}(\mu \cdot n)$, although it also depends on the other two parameters, H and r_1.

5 CONCLUSIONS

In this paper we have presented a highly efficient generator of self-similar traces based on the M/G/∞ model. The model is flexible enough to deal with a wide range of input parameters. We use the decomposition property of Poisson process and the memoryless property of geometric random variables, in order to minimize the mean number of random values to be generated for each sample value of the occupancy process. We have checked both analytically and experimentally the efficiency of the simulator, being the results very satisfactory.

APPENDIX A: IMPLEMENTATION NOTES

We have completed the C++ class `Cox`, that follows the interface of the class `Random` of the library *GNU libg++* as a guideline. We have attempted to provide an implementation as efficient as possible, intending to have approximately the same level of efficiency as in a generator of any random variable.

An object of class `Cox` has the following member objects:

`servers` An object of class `ListTimes` which stores the number of users in the system, their departure times (in slots) and the isochronous clock.

`deter_batches` An array of K objects of class `IntPoisson` which generate samples of Poisson random variables with mean values $\lambda \cdot \mathsf{d}_i$ for $i = 1,2,\ldots,\mathsf{K}$.

`geom_batches` An array of L objects of class `IntPoisson` which generate samples of Poisson random variables with mean values $\lambda \cdot \mathsf{g}_i^{'}$ for $i = 1,2,\ldots,\mathsf{L}$.

`batch` An object of class `IntPoisson` which generates samples of a Poisson random variable with mean value $\lambda \cdot \mathsf{r}$.

`geom_demands` An array of L objects of class `Binomial` which generate samples of the binomial random variables B_i for $i = 1,2,\ldots,\mathsf{L}$.

`demand` An object of class `IntPareto_U_R` which generates samples of the random variable R.

In order to store for each shifted geometric random variable $\mathrm{G}_i^{'}$ the parameter p_i and the composition factor $\mathsf{g}_i^{'}$, the class `Cox` includes two arrays, `geom_params` and `geom_comp_factors`, of size L.

Moreover, since the range of the shifted geometric random variables begins at $k = \mathsf{K}+1$ another array, `delays`, of size $(\mathsf{K}+1)\cdot\mathsf{L}$ is needed in order to delay the departures $\mathsf{K}+1$ units of time.

IntRandom

Both classes `IntPareto_U_R` and `IntPoisson` are built from the class `IntRandom`, which implements a generic tabular method to invert the distribution function of a nonnegative discrete random variable (Suárez et al. 2002).

Binomial

We use the BTRD algorithm combined to the BIN algorithm (Hormann 1993) to generate binomial random values, because the resulting algorithm is very fast, has small memory requirements and is especially suitable for the case that the parameters are random values themselves, which are computed in execution time.

APPENDIX B: S DISTRIBUTION

Considering two separate intervals for the parameter m, the distribution of the S discrete-time random variable $\Pr[\mathrm{S} = k]$ is for $\mathsf{m} \le 1$:

- $1 + \dfrac{\mathsf{m}^\alpha}{\alpha\mathsf{m} - \mathsf{m}^\alpha} \cdot \left[(k+1)^{1-\alpha} - k^{1-\alpha}\right]$
  $k = 1$
- $\dfrac{\mathsf{m}^\alpha}{\alpha\mathsf{m} - \mathsf{m}^\alpha} \cdot \left[(k+1)^{1-\alpha} - 2\cdot k^{1-\alpha} + (k-1)^{1-\alpha}\right]$
  $\forall k > 1$,

and for $\mathsf{m} > 1$:

- $1 + k - \mathsf{m} + \dfrac{\mathsf{m}^\alpha}{\alpha - 1} \cdot \left[(k+1)^{1-\alpha} - \mathsf{m}^{1-\alpha}\right]$
  $k = \lfloor \mathsf{m} \rfloor$
- $1 + \mathsf{m} - k + \dfrac{\mathsf{m}^\alpha}{\alpha - 1} \cdot \left[(k+1)^{1-\alpha} - 2k^{1-\alpha} + \mathsf{m}^{1-\alpha}\right]$
  $k = \lceil \mathsf{m} \rceil$
- $\dfrac{\mathsf{m}^\alpha}{\alpha - 1} \cdot \left[(k+1)^{1-\alpha} - 2k^{1-\alpha} + (k-1)^{1-\alpha}\right]$
  $\forall k > \lceil \mathsf{m} \rceil$.

In Figure 9 we show the form of the distribution for several values of the parameters H and r_1.

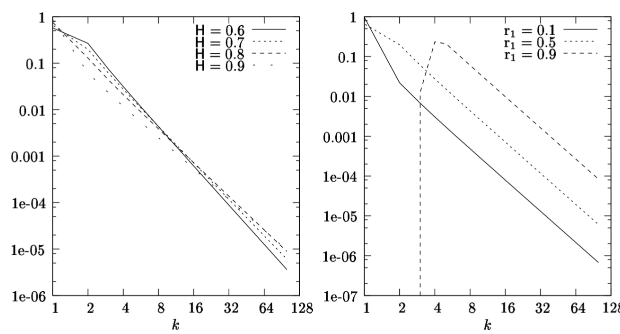

Figure 9: Distributions of S for $\mathrm{r}_1 = 0.5$ (Left) and $\mathsf{H} = 0.7$ (Right)

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