## MULTI-LOCATION TRANSSHIPMENT PROBLEM WITH CAPACITATED PRODUCTION AND LOST SALES

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## ABSTRACT

We consider coordination among stocking locations through replenishment strategies that explicitly take into account lateral transshipments, i.e., transfer of a product among locations at the same echelon level. The basic contribution of our research is the incorporation of supply capacity into the traditional emergency transshipment model. We formulate the capacitated production case as a network flow problem embedded in a stochastic optimization problem. We develop a solution procedure based on infinitesimal perturbation analysis (IPA) to solve the stochastic optimization problem numerically. We analyze the impact on system behavior and on stocking locations' performance when the supplier may fail to fulfill all the replenishment orders and the unmet demand is lost. We find that depending on the production capacity, system behavior can vary drastically. Moreover, in a production-inventory system, we find evidence that either capacity flexibility (i.e., extra production) or transshipment flexibility is required to maintain a certain level of service.

# **1 INTRODUCTION**

Physical pooling of inventories has been widely used in practice to reduce cost and improve customer service. However, information pooling, which entails the sharing of inventory among stocking locations through lateral transshipments, has been less frequent. Transshipments, the monitored movement of material between locations at the same echelon, provide an effective mechanism for correcting discrepancies between the locations' observed demand and their available inventory. As a result, transshipments lead to cost reductions and improved service without necessarily increasing system-wide inventories. In our current research, we focus on collaborative planning and replenishment policies via information pooling and, in particular, on transshipments as a way to improve both cost and service. Our study is motivated by observations from various industries. Inventory-pooling strategies to hedge against the risk of supply disruption are quite common in retailing. Different retail stores or dealerships pool inventory of their products to increase the effectiveness of their safety stock. Container shipping lines also pool their containers through an exchange. Transshipments are increasingly common in apparel, fashion goods, and toys, particularly by those retailers with brick and click outlets. Transshipments of spare parts within a network of plants are quite frequent as well.

All these transshipment practices, however, represent a *reactive* approach to unexpected stockouts. We believe that, if we take transshipment opportunities into account during the planning phase, they can work as an effective mechanism for reducing cost and improving service. Therefore, our approach will be to plan the replenishment policy *proactively* considering the existing transshipment option as secondary supply during the review period.

The literature on transshipments has generally addressed either problems with two retailers, e.g., Tagaras (1989), Tagaras and Cohen (1992), Robinson (1990) and Herer and Rashit (1999), or problems with multiple identical retailers, e.g., Krishnan and Rao (1965), Jönsson and Silver (1987), and Robinson (1990). In contrast, we consider multiple retailers, who may differ both in their cost structures and in their demand parameters, as in Herer, Tzur, and Yücesan (2006). We further consider a supplier with limited production capacity as in Jönsson and Silver (1987), who aim to minimize the total expected end-ofperiod backorders at stocking locations by using transshipment before observing demand to reassure an adequate inventory allocation. In our setting, the total amount of product supplied to N locations is restricted for each time period. When total replenishment orders exceed total supply, not all locations will be fully replenished. Therefore, a rule for the allocation of the available quantity among stocking locations must also be specified. Other recent

work on transshipments includes Archibald et al. (1997), Bertrand and Bookbinder (1998), Tagaras (1999), Rudi et al. (2001), Herer and Tzur (2001, 2003), Slikker et al. (2004), Bendoly (2004), Dong and Rudi (2004), and Wong et al. (2005).

We propose a simulation-based optimization approach for solving the multi-location transshipment problem with supplier capacity and lost sales. To minimize the total system costs, the objective is to find the appropriate inventory policies, which are typically a base stock order-up-to policy. Given a modified order-up-to-*S* policy, we use a network flow framework to determine a myopically optimal transshipment policy between any pair of stocking locations.

Simulation-based derivative estimates help the search for an improved policy while allowing for complex features that are typically outside of the scope of analytical models. Infinitesimal perturbation analysis (IPA) is an efficient simulation-based optimization technique (Ho et al. 1979). With IPA, instead of using finite differences in a gradient search method, we use the mean value of the sample path derivative, which is obtained through a single simulation. Glasserman (1991) established the general conditions for the unbiasedness of the IPA estimator. Applications of IPA have been reported in simulations of Markov chains (Glasserman 1992), inventory models (Fu 1994), manufacturing systems (Glasserman 1994), finance (Fu and Hu 1997), and control charts for statistical process control (Fu and Hu 1999). IPA-based methods have also been introduced to analyze supply chain problems (Glasserman and Tayur 1995, Herer et al. 2006).

The remainder of the paper is organized as follows: In the following section, we introduce the capacitated transshipment problem and the notation used in the paper. Section 3 is devoted to determining the replenishment quantities incorporating various allocation rules under limited supplier capacity. The policy for replenishments and transshipments together with the formulation is explained in Section 4. Section 5 presents the details of the solution technique. We illustrate the solution technique with a numerical study and discuss the findings in Section 6. We conclude with final remarks in Section 7.

## 2 THE MODEL

We consider a supplier serving N retailers, or stocking locations, which face random customer demand. The demand distribution of each stocking location in a period is assumed to be known and stationary over time. The stocking locations review their inventory periodically and replenishment orders are placed with the supplier that has a *finite* total production capacity,  $C^{prod}$ . In any period, transshipments provide a means to reconcile demand-supply mismatches. Within each period, events occur in the following order: the first event in each period is the arrival of replenishment orders placed in the previous period. These orders are used to increase inventory. Next in the period is the occurrence of demand. Since demand represents the only uncertain event of the period, once it is observed, all the decisions of the period, namely, the determination of the transshipment and replenishment quantities, are taken. The transshipment transfers are then made immediately, and subsequently the demand is satisfied. Unsatisfied demand is lost. At this point, inventories and lost demand are observed, and holding and penalty costs, respectively, are incurred. The remaining inventory is carried to the next period.

We consider *modified* base stock policies for replenishment. The policy is "modified" in the following sense. In a base stock policy, when the supplier does not have a capacity constraint, the inventory positions at all stocking locations are raised up to  $S_i$  units at the beginning of each period. Given the finite supplier capacity, however, the locations may not receive the full replenishment quantity ordered in the previous period. Therefore, order-up-to levels may not be attained at the beginning of each period. When replenishment through the supplier is capacitated, different allocation rules are considered to specify how the supplier rations its limited capacity among the locations.

#### 2.1 Notation

In developing our model, we use the following parameters:  $c_i =$  unit procurement cost at stocking location *i*;

- $\hat{t}_{ij}$  = direct transshipment cost per unit transshipped from stocking location *i* to stocking location *j*; this is the additional administrative and logistics costs (packaging, re-labeling, transferring, etc.) per unit due to transshipment.
- $t_{ii}$  = effective transshipment cost, or simply transshipment
  - cost, per unit transshipped from stocking location *i* to stocking location *j*,  $t_{ij} = \hat{t}_{ij} + c_i c_j$ ;
- $h_i$  = holding cost incurred at stocking location *i* per unit held per period;
- $p_i$  = penalty cost incurred at stocking location *i* per unit of lost demand per period.

We assume (as was assumed in Tagaras (1989), Robinson (1990) and Herer and Rashit (1999) as well as others) the following relationships regarding the problem parameters:

$$\begin{aligned} & h_i < h_j + t_{ij} & i, j = 1, ..., N \\ & p_j < p_i + t_{ij} & i, j = 1, ..., N \\ & t_{ij} < h_i + p_j & i, j = 1, ..., N \end{aligned}$$

In the first relationship, we assume that it is not worthwhile to transship between two locations, each having a surplus. In other words, shipping items to one stocking location is not allowed, if there is already a surplus item there. Similarly, in the second inequality, we assume that it is not worthwhile to transship between two locations, each having a shortage. And finally, we assume that if there is a shortage at one of the stocking locations and surplus at another, lateral transshipments are (myopically), cost advantageous. These inequalities ensure that transshipment from location i to location j is economically justifiable only if location i has excess inventory and location jhas shortage.

In addition, we have

- $D_i$  = random variable associated with the periodic demand at location *i* with  $E[D_i] = \mu_i < \infty$ ;
- $d_i^n$  = actual realization of demand at stocking location *i* in period *n*; when we consider demand in an arbitrary period, time superscripts are dropped.

 $C^{prod}$  = total production capacity per period

$$(C^{prod} > E\left[\sum_{i} D_{i}\right]);$$

 $I_i^n$  = net inventory level at stocking location *i* at the be-

ginning of period *n* after replenishment.  $I_i^n$  is the net inventory level in period *n* after the arrival of replenishment orders from the previous period, but before demand is observed.

 $I_{i0}^n$  = net inventory level at stocking location *i* at the end of period *n*.

Two decisions need to be made for each stocking location every period: Transshipment quantities between any pair of stocking locations and replenishment quantities. The associated decision variables are the following:

 $S_i$  = target inventory level (or order-up-to level) at stocking location *i* at the beginning of each period;

 $F_{B_iM_j}^n$  = number of items transshipped from stocking

location *i* to stocking location *j* in period *n* (this notation is motivated by the network flow formulation in section 4);

 $R_i^n$  = number of items *received* from the supplier by stocking location *i* in period *n*+1 that were ordered from the supplier in period *n*. Note that, when production is capacitated, the number of items received is not necessarily equal to the number of items ordered.

### **3 DETERMINING THE REPLENISHMENT QUANTITIES**

In any period *n*, the net inventory level at stocking location *i* at the end of period  $(I_{i0}^n)$  is the sum of the inventory level in period *n*, immediately after demand is observed

 $(I_i^n - d_i^n)$ , and the difference between the total quantity received (via transshipments from other locations,  $\sum_{j:j \neq i} F_{B_j M_i}^n$ ) and sent (via transshipments to other loca-

tions,  $\sum_{j:j\neq i} F_{B_iM_j}^n$ ) during period *n*. Therefore, in period

n+1, the net inventory level at stocking location  $i(I_i^{n+1})$ immediately before demand  $(d_i^{n+1})$  is observed, is the sum of the on-hand inventory level in period n at the end of period  $n(I_{i0}^{+}(n))$  and items received from the supplier in period  $n(R_i^n)$  In each period, the replenishment quantity  $R_i^n$  is the minimum of remaining production capacity  $(C^{prod} - \sum_{j \neq i} R_j^n)$  and the difference between the order-up-

to value and the inventory level at the end of the period at location i. Therefore, the sample path of the system in any period n can be described as follows:

$$I_{i0}^{n} = I_{i}^{n} + \sum_{j:j \neq i} F_{B_{j}M_{i}}^{n} - \sum_{j:j \neq i} F_{B_{i}M_{j}}^{n} - d_{i}^{n}, \ i = 1, \dots, N$$

$$I_i^{n+1} = I_{i0}^+(n) + R_i^n, \quad i = 1, \dots, N$$
(1)

$$R_i^n = \min\left(\left(C^{prod} - \sum_{j \neq i} R_j^n\right), \left(S_i - I_{i0}^n\right)\right)$$
(2)

and, under complete pooling,  $F_{B_iM_j}^n$ , the transshipment quantity from stocking location *i* to *j* is equal to

$$H_{B_{i}M_{j}}^{n} = \min\left\{I_{i}^{n} - d_{i}^{n} - \sum_{\substack{k \neq j; k \neq i}} H_{B_{i}M_{k}}^{n}\right)^{+} \left(d_{j}^{n} - I_{j}^{n} - \sum_{\substack{k \neq j; k \neq i}} H_{B_{k}M_{j}}^{n}\right)^{+}\right\} \quad i \neq j. \quad (3)$$

The total cost of the system in period *n* is given by:

$$TC_n = \sum_{i=1}^N \sum_{j=1}^N t_{ij} F_{B_i M_j}^n + \sum_{i=1}^N h_i I_{i0}^+(n) + \sum_{i=1}^N p_i I_{i0}^-(n) + \sum_{i=1}^N c_i d_i,$$

where  $I_{i0}^+(n) = \max\{0, I_{i0}^n\}$  and  $I_{i0}^-(n) = \max\{0, -I_{i0}^n\}$ . The unit purchase cost at location *i* is multiplied by the demand at location *i* and not by the replenishment quantity at location *i* since the procurement cost differentials are included in the transshipment costs.

When total replenishment orders exceed total supply capacity, not all locations will be able to attain their base stock levels. We will refer to this difference between the order-up-to level ( $S_i$ ) and the inventory level at location *i* at

the beginning of period n+1 ( $I_i^{n+1}$ ) as the *shortfall* at location *i* at the end of period *n*. We will use the shortfall values later in the analysis. Moreover, for the allocation of the available supplier capacity among stocking locations, we implement four allocation rules: (i) beginning inventory balancing rule; (ii) shortfall balancing rule; (iii) equal allocation rule; (iv) priority-based allocation rule.

## 4 DETERMINING THE TRANSSHIPMENT QUANTITIES

In each period, the replenishment and transshipment quantities must be determined. Herer et al. (2006), who focused on the uncapacitated version of our problem, proved that, if transshipments are only made to compensate for an actual shortage (instead of building up inventory at another stocking location), there exists an optimal order-up-to  $S = (S_1, S_2, ..., S_N)$  policy for all possible stationary transshipment policies. For the capacitated case, their characterization of the optimal replenishment policy is an open problem. Nevertheless, since the transshipment policy is stationary and the fixed ordering cost is negligible, we will continue to adhere to an order-up-to *S* replenishment policy.



Figure 1: Network Flow Representation of the Problem for Period n

Once demand is observed, for a given base stock level, it is possible to solve the transshipment decision problem via a network flow formulation. After solving for the myopically optimal transshipment decision, instead of using  $R_i$  values obtained through the LP, we will make the replenishment decision in accordance with one of the allocation rules.

We adapt the complete network flow problem and the associated LP formulation approach of Herer et al. (2006) to model the multi-location capacitated transshipment problem. The network flow representation of the problem is illustrated in Figure 1 for a 3-retailer configuration. To keep the notation consistent with their paper, we will use the following decision variables in our LP formulation:

 $F_{B_iM_j}^n$ : transshipment quantity from stocking location *i* to stocking location *j* in period *n*;

 $F_{B_i E_i}^n$ : inventory held at stocking location *i* in period *n*, which is denoted by  $I_{i0}^+(n)$  in Section 3;

 $I_i^-(n)$ : shortage at stocking location *i* in period *n*, which translates into lost sales;

 $F_{RE_i}^n$ : total replenishment for stocking location *i* in period *n*;

 $I_i^+(n+1)$ : on-hand inventory at stocking location *i* in the beginning of period n+1, after arrival of replenishment orders.

In the network flow problem formulation (as depicted in Figure 1), we have a source node,  $B_i$ , to represent the <u>beginning</u>, i.e., initial inventory at stocking location *i*, after replenishment orders arrive, and a source node, *R*, to represent the <u>replenishment</u> that is requested in the period but arrives at the start of the next period. The sink node associated with the demand at retailer *i* is denoted by  $M_i$ . Similarly, we denote by  $E_i$  the <u>ending</u> inventory at stocking location *i*, including units on order from the supplier. We use the letter 'F' to denote the <u>f</u>lows in the network and subscripts to indicate the starting and ending nodes of the flows; thus,  $F_{B_iM_j}^n$  is the flow in the network from node  $B_i$  to  $M_i$  in period *n*.

### 4.1 LP Formulation with Lost Sales

Thus, the formulation for the case where unsatisfied demand is lost is as follows:

$$(\vec{P}_{n})$$
min  $TC = \left(\sum_{i=1}^{N} \sum_{j=1}^{N} t_{ij} F_{B_{i}M_{j}}^{n} + \sum_{i=1}^{N} h_{i} F_{B_{i}E_{i}}^{n} + \sum_{i=1}^{N} p_{i} I_{i}^{-}(n)\right)$ 

subject to

$$\sum_{i=1}^{N} F_{B_{i}M_{j}}^{n} + F_{B_{i}E_{i}}^{n} = I_{i}^{+}(n), \ i = 1, ..., N$$
(4)

$$I_i^+(n+1) \le S_i \quad i = 1, ..., N$$
 (5)

$$\sum_{i=1}^{N} F_{RE_i}^n \le C^{prod}, \tag{6}$$

$$\sum_{j=1}^{N} F_{B_j M_i}^n + I_i^-(n) = d_i^n, \ i = 1, ..., N$$
<sup>(7)</sup>

$$F_{B_i E_i}^n + F_{R E_i}^n = I_i^+ (n+1), \quad i = 1, ..., N$$
(8)

$$I_i^+(n+1) \ge 0, \quad i = 1, ..., N$$
 (9)

As defined earlier, let  $I_i^-(n)$  denote the unsatisfied demand of location *i* in period *n*, which will be lost, and  $p_i$  be the unit penalty cost of lost sales at location *i*. Then, in the objective function, the expression  $(\sum_{i=1}^{N} p_i I_i^-(n))$  can be interpreted as the total penalty for lost sales. The constraint sets (4) and (8) ensure the balance of the inventory position of each stocking location at the beginning and at the end of each period, respectively. Constraint set (7) guarantees that the observed demand at location *i* (*d<sub>i</sub>*) will be satisfied either from the location's own inventory  $(F_{B_iM_i}^n)$ , transshipped from another location  $(F_{B_jM_i}^n)$  or lost  $(I_i^-(n))$ . Moreover, due to the supplier capacity constraint, the in-

Moreover, due to the supplier capacity constraint, the inventory position may not attain the order-up-to levels,  $S_i$ , which is captured by constraint set (5). Finally, constraint (6) guarantees that total replenishment to all stocking locations will be at most  $C^{prod}$  units, reflecting supplier capacity. Non-negativity constraints (9) are also included. The objective is to minimize the cost of demand-supply mismatch (inventory holding and shortage penalty costs) and transshipment costs.

## 5 THE SOLUTION ALGORITHM

For the capacitated transshipment problem, determining the exact order-up-to levels is analytically difficult. To compute the order-up-to-S values, we therefore use a samplepath-based optimization procedure, based on IPA, to minimize the total average cost per period. In IPA, the idea is to use the expected value of the sample path derivative obtained via simulation, instead of using the derivative of the expected cost, in a gradient search algorithm to update the S<sub>i</sub> values. In particular, we start with arbitrary order-up-to levels,  $S_i$ , for each stocking location. After randomly generating an instance of the demand at each stocking location, an LP formulation is constructed in a deterministic fashion to compute the transshipment quantities, i.e., to solve problem  $(\widetilde{P}_n)$ . The solution of the LP provides us with a set of replenishment quantities. These replenishment quantities reflect the allocation of limited supplier capacity under the priority based allocation rule. However, since we want to experiment with various different allocation rules, we determine the replenishment quantities and the gradient values for that particular period based on the replenishment allocation rules introduced in Section 3.

The fact that, for a linear program, the dual value of a constraint is the derivative of the objective function with respect to the right-hand side of that constraint is already used by Swaminathan and Tayur (1999). To calculate the gradient values in our setting, however, we need to go further. In particular, we need to calculate the gradient over one complete regenerative cycle, two consecutive periods in which all stocking locations simultaneously reach their order-up-to levels,  $S_i$ , which is often more than a single period. Therefore, we need to propagate the gradients through the periods in the cycle. Moreover, since we do not use the replenishment quantities  $(R_i)$  calculated by the LP, the shadow prices provided by the sensitivity analysis output of the LP solution will not be helpful. As a consequence, we use the LP to compute the transshipment quantities, but not the replenishment quantities; we further use the LP output to accumulate the IPA gradients  $(\partial TC / \partial S_i)$ , which are used in a path search algorithm to determine the optimal order-up-to levels. This is indeed the specialization of stochastic approximation to the capacitated transshipment problem. The stochastic optimization algorithm outlined in Figure 2 exploits this property; the technical details are presented in Özdemir (2004).

When supply capacity is limited, however, it may take several periods of supply to fulfill all replenishment requests received in a single period. Therefore, we need to establish that the number of periods to fulfill all replenishment requests (i.e., the length of the regenerative cycle) is finite so that eventually all locations reach their order-up-to levels. In other words, we need to establish the stability of the replenishment policy. This is also done in Özdemir (2004).

### **6 COMPUTATIONAL STUDY**

We analyze the impact of different factors on transshipment relations with limited production capacity under 5 configurations of 10 stocking locations. In all configurations, we consider stocking locations with identical cost parameters. In particular, we set the holding cost to  $h_i =$ \$ 1 and penalty cost to  $p_i =$ \$ 4 for all ten locations. Each location faces an independent demand distributed uniformly over (0, 200). As summarized in Table 1, we consider five alternative system configurations with different unit transshipment costs,  $t_{ij}$ , for units transshipped from stocking location *i* to stocking location *j*. Note that  $t_{ij} = \infty$  implies that transshipments are not allowed between locations *i* and *j*.

As a base case, in system #1, no material movement is allowed among stocking locations, turning the system into 10 independent newsvendors. In system #2, only the first stocking location can transship to the other stocking locations. In system #3, transshipments from all stocking loca-



Figure 2: Basic Modules of the Solution Algorithm

Table 1: System Configurations

	<u> </u>		
System	$t_{1i}$	$t_{i1}$	t <sub>ij</sub>
1	~	8	~
2	0.5	$\infty$	$\infty$
3	0.5	0.5	$\infty$
4	0.5	0.5	1.0
5	0.5	0.5	0.5

tions to the first stocking location are also allowed. In systems #4 and #5, all material movement is allowed. In system #4, however, transshipments between any two stocking locations (which do not include location #1) are twice as expensive. For each system, we generate nine scenarios with different supplier capacities. The capacity values used are:

 $C^{prod}$  = {1050, 1100, 1150, 1200, 1250, 1300, 1400, 1500,  $\infty$ }.

In general, the selection of effective values for algorithm parameters is a difficult problem; we set the total number of steps for the path search to K=2000, the number of regenerative cycles at each step to U=2000, and the step size to  $\alpha_k = 1000/k$ . As a stopping criterion, we compare the computed order-up-to levels over 1000 iterations and require that these values do not differ by more than 1. In all of our experiments, the convergence criterion was satisfied before 2000 steps.

For the lost sale scenarios, we observe a pattern very similar to the ones with backlogged demand. When production capacity increases, total cost of the systems decrease. In particular, switching from system # 1, a configuration without transshipments, to system # 2, a con-



figuration with limited transshipment flexibility, results in large cost savings. We observe that savings through transshipments are more pronounced when unsatisfied demand is lost. We have a similar observation about the total inventory when unsatisfied demand is lost. As can be seen in Figure 3, when unsatisfied demand is lost total inventory carried in any system is relatively stable. We still observe that the inventory levels decrease as the production capacity increases. Nevertheless, this impact is practically insignificant.

## 7 SUMMARY

We consider a supply chain, which consists of N stocking locations and one supplier. The locations may be coordinated through replenishment strategies and lateral transshipments. The supplier has limited production capacity. Therefore, total amount of product supplied to N locations is limited for each time period. When total replenishment orders exceed total supply, not all locations will be able to attain their base stock values. Therefore, different allocation rules are considered to specify how the supplier rations its limited capacity among the locations. Unmet customer demand is lost. We team up the modeling flexibility of simulation with stochastic optimization to address the multi-location transshipment problem. With this numerical approach, we can study problems with non-identical costs and correlated demand structures.

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