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EFFICIENT SIMULATIONS OF SUPPLY CHAINS

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ABSTRACT

High volume production flows are modeled by nonlinear hyperbolic partial differential equations representing conservation laws. These models have Little's law explicitly built into the formulation. Borrowing from concepts in gas dynamics and vehicular traffic models we derive several prototypical equations representing linear as well as re-entrant factories. Multiple products, dispatch polices and control actions can be modeled. Standard hydrodynamic codes provide very fast simulations of these models allowing us to link them together to form efficient supply chain simulations.

1 INTRODUCTION

Understanding the behavior of large supply chains under different polices and scenarios is a major issue for many businesses today. Obviously there are no controlled experiments that can be done involving whole supply chains or even involving only a single large factory. Hence simulation models will have to be developed that substitute for the real environment. While discrete event simulators have been highly successful to simulate single factories, for instance simulations of semiconductor manufacturing at the tool level, they are much too computationally expensive to simulate even a moderately complicated supply chain. Alternative models that endow supply chain nodes with fixed production capacities and fixed lead times are not accurate enough since they do not take into account the fact that capacitated system respond nonlinearly to increases in demand close to the limit of the production capacity. In this paper we propose continuum models of factory production that treat the flow of products much like a traffic flow or a hydrodynamic flow. We discuss models for a linear factory and a re-entrant factory, typical for semiconductor production. The resulting models are nonlinear, nonlocal, hyperbolic conservation laws. There are standard numerical algorithms that allow a very fast and efficient simulation of such equations. By linking three factories together to a supply chain we demonstrate the feasiblilty of our approach.

2 HYPERBOLIC CONSERVATION MODELS

Real factories show a strong increase in the average throughput time τ as the loading of the factory is increased. Unfortunately, large factories are rarely run in equilibrium for any amount of time and are too costly to be run as a controlled experiment. Hence the specific nature of the throughput time nonlinearity is unclear. We have developed two models that can serve as extreme case models for real factories. Most likely any real factory will behave in some intermediate manner. The two models roughly correspond to a linear factory where every production step has its own machine and a re-entrant factory where product has repeated passes through the same machine, respectively. We call the former a queuing model and the latter a re-entrant model.

Both models are based on the fundamental fact of conservation of jobs: whatever enters the factory has to come out of the factory at some time (we neglect the yield issue for the moment). We define x to be a "completion" variable with $x = 0$ denoting the start of a product into the factory and $x = 1$ the release of a finished product. Writing $\rho(x, t)$ for the density of work at stage x at time t we get the total load (WIP) as a function of time

$$L(t) = \int_0^1 \rho(x, t) dx.$$

The density then satisfies a hyperbolic conservation law of the form

$$\rho_t + (v(\rho)\rho)_x = 0 \qquad (1)$$

where $v(\rho)$ describes the velocity of product moving in the factory. In general, $v(\rho)$ will have a functional, non-local dependence on ρ as opposed to a simple local function of ρ as in a traffic flow model. The exact nature of the transport velocity $v(\rho)$ is the major modeling issue.

For a re-entrant factory we assume that $v(\rho)$ is described by a state equation of the form

$$v(\rho) = v_0(1 - \frac{L}{L_{max}}). \tag{2}$$

Here v_0 is the speed for the empty factory and L_{max} is the maximal load. Equation (2) implies that the velocity is uniform in the whole factory and that, due to the re-entrant nature of the flow, the total WIP determines this velocity. Notice that v is time dependent through $L(t)$. The start rate $\lambda(t)$ into the factory enters as the boundary condition for the flux at $x = 0$:

$$\lambda = \rho(0,t)v(t). \tag{3}$$

It is easy to see that for any equilibrium solution ρ_{eq} as well as for long term averages (if they exist) equations (1-3) satisfy Little's law (Little 1961): Assume $\rho = \rho_{eq}$. Then $\rho(0,t) = \rho_{eq}$ and with $v = v_{eq} = \frac{1}{\tau}$, equation (3) becomes Little's law. The full model (1-3) in essence describes the dynamics of the factory flow as if it were always in equilibrium, following adiabatically the state equation (2).

In contrast to the re-entrant model, a more sophisticated model for the queuing model can be derived (Ringhofer and Armbruster 2002). It allows for non-adiabatic relaxation of the velocity fields: Consider a job arriving at a queue with processing rate μ. Its throughput time depends on the number of jobs waiting before it

$$\tau = \frac{1}{\mu}(1 + L). \tag{4}$$

Using this relationship as a boundary condition we derive a set of coupled hyperbolic conservation laws for the WIP density $\rho(x,t)$ and the velocity $v(x,t)$ of the form:

$$\begin{aligned} \rho_t + (v\rho)_x &= 0 && (5)\\ v_t + v v_x &= 0 && (6)\\ v(0,t) &= \frac{\mu}{1 + L(t)} && (7)\\ v(0,t)\rho(0,t) &= \lambda(t). && (8) \end{aligned}$$

The heuristic model for the re-entrant flow can be extended to include many issues relevant to a real simulation:

- Multiple products: To model production of two products, A and B, we extend our formulation to two densities ρ_A and ρ_B and two state equations

$$\begin{aligned} v_A &= v_0^A(1 - \tfrac{L}{L^A_{max}})\\ v_B &= v_0^B(1 - \tfrac{L}{L^B_{max}})\\ L &= \textstyle\int_0^1 (\rho_A + \rho_B)ds \end{aligned} \tag{9}$$

which allows for different raw speeds of the A and B product and for different capacities of the factory relative to the two products.
- Dispatch rules: To model a rule of preference of product A over product B we put weight functions on the load: For instance, if we choose product A over product B, then product B will not influence the speed of A through the factory, hence

$$\begin{aligned} L^A(t) &= \textstyle\int_0^1 \rho_A ds\\ L^B(t) &= \textstyle\int_0^1 (\rho_A + \rho_B)ds. \end{aligned} \tag{10}$$

Rules based on a specific ratio of As and Bs can be accomplished by having appropriate weight factors attached to the densities of the loads of equation (10).
- Policies: As written so far, our models implicitly assume FIFO policies. However other policies can be implemented using integration kernels $w(x,s)$ that indicate the importance of product at location s in completion space on the speed of product at location x by writing

$$v(x,\rho) = v_0(1 - \frac{1}{L_{max}}\int_0^1 w(x,s)\rho(s)ds. \tag{11}$$

As a result, the velocity will cease to be uniform throughout the factory. For instance, a pull policy is modeled by the kernel

$$\begin{aligned} w(x,s) &= 0 \quad \text{if } s < x,\\ &= 1 \quad \text{if } x \le s. \end{aligned}$$

This leads to

$$v(x,t) = v_0(1 - \frac{1}{L_{max}}\int_x^1 \rho(s)ds. \tag{12}$$

This implies that $v(1) = v_0$, indicating that product at the end of production moves independently of the load of the factory, while $v(0)$ shows the full impact of the loading of the factory on the motion at the beginning of the production line. Similarly, a push policy can be easily implemented. The same idea will also allow for a more detailed modeling of the topology of the factory where the weight function w could in principle account for the impact of the exact re-entrant flow inside the factory.

3 EXPERIMENTS

Before we couple these factories to supply chains we can study the properties of each individual model. A short report on equilibria, their stability and control issues has been given in Marthaler, Armbruster, and Ringhofer (2002); more detail can be found in the thesis of Dan Marthaler (Marthaler (2002)). A comprehensive report is in preparation (Armbruster, Marthaler and Ringhofer 2002). Figure 1 shows some simulations that indicate that the model is functioning properly: We plot the transient throughput times of product moving through a re-entrant factory for increasing start rates. The influx increases from bottom to top according to the inset. The critical start rate is $\lambda = 2.5$. For higher start rates there does not exist an equilibrium solution any more. In addition to the throughput time increasing nonlinearly with the start rate, we also see that the length of the transient increases with increasing λ.

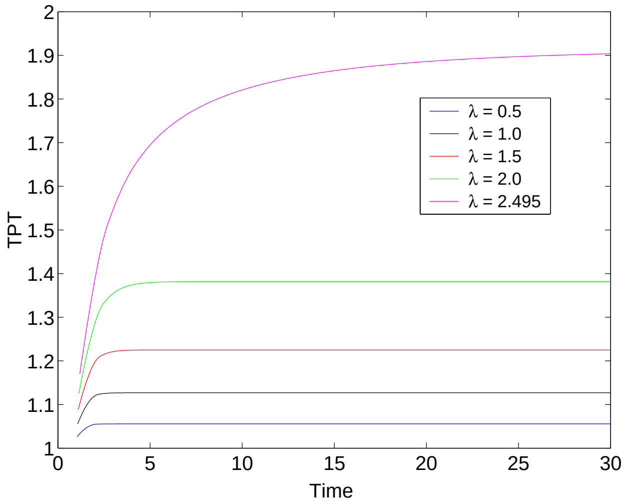

Figure 1: Throughput Time for a Simulation of a Re-entrant Model (Equation (1) and (2) for Varying Start Rates)

While the possibility of shock waves and the conservation law properties of equation (1) require some caution in simulating these equations, there are nevertheless standard numerical codes developed mainly for hydrodynamic purposes (LeVeque 1992) that allow basically instantaneous simulations of these equations. Connecting several nodes, each simulated with an appropriate conservation law formulation, is done in the simplest way via flux coupling: Whatever comes out of one factory has to go into the next factory. This implies that the density will be discontinous along a supply chain but the flux will be a continous variable. Splitting and merging of production streams is also straightforward: Fluxes are added when multiple factories feed into one factory and the outflux of a factory can be split according to any desired policy to feed it into multiple factories downstream. Figure 2 shows a snapshot of a 3 node supply chain: We are simulating a linear factory feeding a re-entrant factory feeding another linear factory. Only one product is simulated and FIFO is used throughout. Each node makes up one third of the completion space (x-axis). The influx into the first factory is a steady periodic function. The snapshot in Figure 2 is taken during a transient generated by starting with empty factories. The re-entrant factory has a much shorter transient and is in steady state while the linear factories are still showing transient behavior. This is a reflection of the fact that we have chosen an average influx that is just below the capacity of the linear factory, with oscillations which during part of the period exceed that capacity, whereas the capacity of the re-entrant factory has been set much higher. In addition, the re-entrant factory acts as a damper. Not only is the variation of WIP and flux lower in the re-entrant part, it is also lower downstream from the re-entrant factory. Figure 3 shows WIP as a function of time for this simulation. The fast transient re-entrant factory, extremly long transients in the linear factories as well as the reduced WIP downstream can be seen.

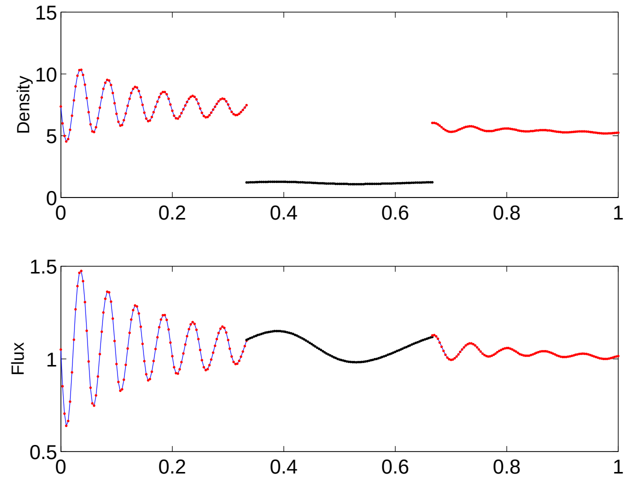

Figure 2: Density and Flux for a 3 Node Supply Chain

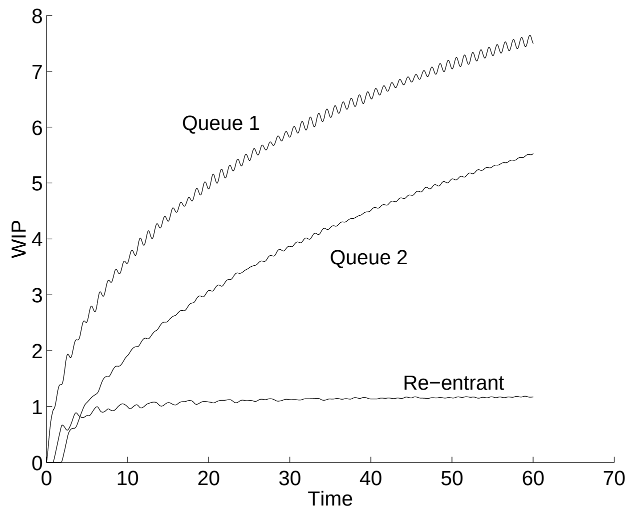

Figure 3: WIP in Each of the Three Nodes as a Function of Time

Clearly series of experiments analyzing the impact of changing capacities along the line, changing input fluxes, etc. on WIP, outflux and troughput time as a function of time can be performed.

Merging such simulations with control concepts may allow us to reduce variability along a supply chain: Assume we have a supply chain where the product flow splits at the output of one factory into two streams that go through two identical factories. The outflux of the two factories is then merged into one input stream. If our flux is periodic (or has a dominant period), we can generate a phase shift by running one factory slower than the other. Alternatively, the throughput time through these factories will be very susceptible to the total load inside the factory. Hence by splitting the flux exactly 50-50 but adding a second product into one of the factories we can create a load difference between the two factories. If done correctly this will lead to a phase shift that may be used to dampen the oscillations of the merged flux. We expect to have simulations available of these ideas for the conference.

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